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SHRINKAGE TESTIMATORS OF SCALE PARAMETER FOR EXPONENTIAL MODEL UNDER ASYMMETRIC LOSS FUNCTION

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Abstract

The present paper proposes shrinkage testimator(s) for the scale parameter for an exponential distribution. An important feature of the proposed testimator is that, it removes the arbitrariness in the choice of shrinkage factor (weights) by making it dependent on the test statistic. The risk properties of the proposed testimator(s) have been studied under asymmetric loss function. It has been observed that the proposed testimator performs better than the classical Uniformly Minimum Variance Unbiased Estimator (UMVUE). Recommendations regarding its applications for various degrees of asymmetry (over/under estimator), level(s) of significance have been made.

Key words: Exponential distribution, scale parameter, preliminary test, level of significance, asymmetric loss function, relative risk.

1. Introduction

The Exponential distribution has been a subject of comprehensive studies since early fifties. A systematic development of life testing originated from the work of Epstiem and sobel (1953) and the subsequent progress made in this field can be gauged from the bibliography of Mendenhall (1958) and Govindrajulu (1964), among many others.

Several authors have proposed various estimators/testimators of exponential scale parameters/ testimators of exponential scale parameter using different loss functions, mainly using the Squared Error Loss Function (SELF) the mean square error have been obtained and attempts were made to minimize the MSE and to propose minimum mean square estimator (MMSE).

Another approach of shrinkage estimation along the lines of Thompson (1968) has been studied by several authors. While proposing shrinkage estimators/ testimators, the shrinkage factor plays an important role. Estimators of this type with ‘k’ (the shrinkage factor) arbitrary ($0 \leq k \leq 1$) have been defined and studied in different contexts by several authors such as Bhattacharya and Srivastava (1974), Pandey (1983), Pandey and Srivastava (1987) among others.

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Srivastava (1987) has proposed a shrinkage testimator of scale parameter in exponential distribution taking ‘k’ dependent on test statistics and, the arbitrariness in the choice of ‘k’ has been removed. There could be several other choice of ‘k’. Properties of this estimator have been studied and recommendations are made.

The present paper deals with proposing the shrinkage testimator and studying its properties under asymmetric loss function. It has been observed that in many estimation problems, the use of SELF may be in appropriate as has been pointed out by Canfield (1970), Varin (1975), Zellner (1986), Basu and Ebrahimi (1991), Srivastava and Tanna (2001) have considered an estimation procedure for error variance in corporating PTS under LINEX Loss Function.

Varian (1975) proposed asymmetric loss function, which has been found to be appropriate in the situations where overestimation is more serious than underestimation or vice-versa.

While estimating a parameter θ by $\hat{\theta}$, this loss function is given by,

$$L(\Delta) = b[e^{a\Delta} - a\Delta - 1],\ b > 0,\ a \neq 0 \qquad (1.1)$$

Where $\Delta = \left(\frac{\hat{\theta}}{\theta} - 1\right)$

The sign and magnitude of 'a' represents the direction and degree of asymmetry respectively. The positive value of 'a' is used when overestimation is more serious than under estimation, while a negative value of 'a' is used in reverse situations. L(Δ) rises exponentially when Δ < 0 and almost linearly when Δ > 0. The loss function defined by (1.1) is known as the LINEX loss function. 'b' is the factor of proportionality.

In section -2, we have proposed the shrinkage testimator(s). The third section deals with the derivation of risk of the proposed testimator(s) under asymmetric loss function. In the fourth section, we have compared the risk of UMVUE and the shrinkage testimator(s) for the scale parameter. We state our conclusions in section 5.

2. Shrinkage Testimator(s)

Let x have the distribution $f(x;\theta) = \left(\frac{1}{\theta}\right)e^{-x/\theta}$, x ≥ 0, θ > 0 (2.1)

It is assumed that the prior knowledge about θ is available in the form of an initial estimate θo. We are interested in constructing an estimator of θ possibly using the information about θ and the sample observations: x_1, x_2, ----------, x_n. The proposed shrinkage testimator can be described as follows:

(i) Compute the sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

which is the 'best' estimator of θ in absence of any information about θ.

(ii) Test the hypothesis Ho: θ = θo against H_1: θ ≠ θo at α level using the test statistic $\frac{2n\bar{x}}{\theta_0}$ which follows χ^2 – distribution with 2n degrees of freedom.

We define the shrinkage testimator $\hat{\theta}_{ST_1}$ of θ as follows:

$$\hat{\theta}_{ST_1} = \begin{cases} k\bar{x} + (1-k)\theta_0, & \text{if Ho is accepted. } \chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2 \\ \bar{x}, & \text{Otherwise} \end{cases} \qquad (2.2)$$

We observe that ‘k’ defined in (2.2) can take any value between ‘0’ and ‘1’. However, it may be noted that the choice of shrinkage factor (weights) for a given level of significance ‘α’ is no longer arbitrary. We know that the test statistic for testing H_0: $\theta = \theta_0$ in (2.1) is $\frac{2n\bar{x}}{\theta_0}$ which follows χ^2 – distribution with 2n degrees of freedom. Hence, defining $\hat{\theta}_{ST_1}$ again by taking $k = \frac{2n\bar{x}}{\theta_0\chi^2}$, where $\chi^2 = \chi_1^2 + \chi_2^2$; χ_1^2 and χ_2^2 are the lower and upper critical points of χ^2_{2n}. We propose another testimator θ as $\hat{\theta}_{ST_2}$.

$$\hat{\theta}_{ST_2} = \begin{cases} \left(\frac{2n\bar{x}}{\theta_0\chi^2}\right)\bar{x} + \left[1 - \left(\frac{2n\bar{x}}{\theta_0\chi^2}\right)\right]\theta_0 , & \text{if Ho is accepted} \\ \bar{x} , & \text{Otherwise} \end{cases}$$

3. Risk of Testimator(s)

The risk of $\hat{\theta}_{ST_1}$ under L(Δ) defined by

$$R(\hat{\theta}_{ST_1}) = E[\hat{\theta}_{ST_1} \mid L(\Delta)]$$

$$= E\left[k\bar{x} + (1-k)\theta_0 \Big/ \chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right] \cdot p\left[\chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right]$$

$$+ E\left[\bar{x} \,\middle|\, \frac{2n\bar{x}}{\theta_0} < \chi_1^2 \ \mathbf{r}\ \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \cdot p\left[\frac{2n\bar{x}}{\theta_0} < \chi_1^2 \ \mathbf{r}\ \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \tag{3.1}$$

$$= e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left[\frac{k(\bar{x}-\theta_0)+\theta_0}{\theta}\right]} f(\bar{x})\, d\bar{x}$$

$$- a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left[\frac{k(\bar{x}-\theta_0)+\theta_0}{\theta} - 1\right] f(\bar{x})\, d\bar{x}$$

$$- \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} + e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left(\bar{x}/\theta\right)} f(\bar{x})\, d\bar{x}$$

$$- a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left(\frac{\bar{x}}{\theta} - 1\right) f(\bar{x})\, d\bar{x} - \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} \tag{3.2}$$

Where $\quad f(\bar{x}) = \dfrac{1}{\Gamma n}\left(\dfrac{n}{\theta}\right)^{n}\left(\bar{x}\right)^{n-1} e^{\frac{-n\bar{x}}{\theta}}$

Straight forward integration of (3.2) gives

$$R(\hat{\theta}_{ST_1}) = \left\{ I\left(\frac{\chi_1^2\phi}{2}, n\right) - I\left(\frac{\chi_2^2\phi}{2}, n\right) + 1 \right\}\left\{ \frac{e^{-a}}{\left(1 - a/n\right)^{n}} - 1 \right\} +$$
$$\left[a\left\{ I\left(\frac{\chi_2^2\phi}{2}, n+1\right) - I\left(\frac{\chi_1^2\phi}{2}, n+1\right) \right\}\right] (1-k) +$$
$$\left\{ I\left(\frac{\chi_2^2\phi}{2}, n\right) - I\left(\frac{\chi_1^2\phi}{2}, n\right)\right\}\left\{ \frac{e^{-a} e^{a\phi(1-k)}}{\left(1 - ak/n\right)^{n}} - ak\phi - a\phi - 1 \right\} \tag{3.3}$$

Again, we obtain the risk of $\hat{\theta}_{ST_2}$ under L(Δ) given by

$$R(\hat{\theta}_{ST_2}) = E\,[\hat{\theta}_{ST_2} \,|\, L(\Delta)]$$
$$= E\left[\frac{2n\bar{x}}{\theta_0\chi^2}\left(\bar{x} - \theta_0\right) + \theta_0 \Big/ \chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right] \cdot p\left[\chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right]$$
$$+ E\left[\bar{x} \,\middle|\, \frac{2n\bar{x}}{\theta_0} < \chi_1^2 \ \mathbf{r}\ \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \cdot p\left[\frac{2n\bar{x}}{\theta_0} < \chi_1^2 \ \mathbf{r}\ \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \tag{3.4}$$

$$= e^{-a}\int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left[\frac{\frac{2n\bar{x}}{\theta_0\chi^2}\left(\bar{x}-\theta_0\right)+\theta_0}{\theta}\right]} f(\bar{x})\, d\bar{x}$$
$$- a\int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left[\frac{\frac{2n\bar{x}}{\theta_0\chi^2}\left(\bar{x}-\theta_0\right)+\theta_0}{\theta} - 1\right] f(\bar{x})\, d\bar{x}$$
$$- \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} + e^{-a}\int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left(\bar{x}/\theta\right)} f(\bar{x})\, d\bar{x}$$
$$- a\int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left(\frac{\bar{x}}{\theta} - 1\right) f(\bar{x})\, d\bar{x} - \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} \tag{3.5}$$

Where $f(\overline{x}) = \frac{1}{\Gamma n}\left(\frac{n}{\theta}\right)^n (\overline{x})^{n-1} e^{\frac{-n\overline{x}}{\theta}}$

A straight forward integration of (3.5) gives:

$$R(\hat{\theta}_{ST_2}) = I^* - \frac{2a(n+1)}{\phi(\chi^2)^2}\left\{I\left(\frac{\chi_2^2\phi}{2}, n+2\right) - I\left(\frac{\chi_1^2\phi}{2}, n+2\right)\right\} + \left(\frac{2n}{x^2}+1\right)$$
$$\left[a\left\{I\left(\frac{\chi_2^2\phi}{2}, n+1\right) - I\left(\frac{\chi_1^2\phi}{2}, n+1\right)\right\}\right] + \left\{\frac{e^{-a}}{\left(1-a/n\right)^n} - 1\right\}$$
$$\left\{I\left(\frac{\chi_1^2\phi}{2}, n\right) - I\left(\frac{\chi_2^2\phi}{2}, n\right) + 1\right\} - \left\{I\left(\frac{\chi_2^2\phi}{2}, n\right) - I\left(\frac{\chi_1^2\phi}{2}, n\right)\right\}(a\phi+1) \tag{3.6}$$

Where $I^* = e^{a(\phi-1)} \int_{\frac{\chi_1^2\phi}{2}}^{\frac{\chi_2^2\phi}{2}} e^{a\left[\frac{2t^2}{n\phi\chi^2} - \frac{2t}{\chi^2}\right]} \frac{1}{\Gamma n} e^{-t} t^{n-1}\, dt$

4. Relative Risk(s)

A natural way of comparing the risk of the proposed testimators, is to study its performance with respect to the best available estimator $\overline{x}$ in this case, which is also the UMVUE. For this purpose, we obtain the risk of $\overline{x}$ under L(Δ) as:

$$R_E(\overline{x}) = E[\overline{x} \mid L(\Delta)]$$
$$= e^{-a}\int_0^\infty e^{a\left(\overline{x}/\theta\right)} f(\overline{x})\, d\overline{x}$$
$$- a\int_0^\infty \left(\frac{\overline{x}}{\theta} - 1\right) f(\overline{x})\, d\overline{x} - \int_0^\infty f(\overline{x})\, d\overline{x} \tag{4.1}$$

A straightforward integration of (4.1) gives

$$R_E(\overline{x}) = \frac{e^{-a}}{(1-a/n)^n} - 1 \tag{4.2}$$

Now, we define the Relative Risk of $\hat{\theta}_{ST_1}$ with respect to $\overline{x}$ under L(Δ) as follows –

$$RR_1 = \frac{R_E(\overline{x})}{R(\hat{\theta}_{ST_1})} \tag{4.3}$$

Using (4.2) and (3.2) the expression for RR_1 given in (4.3) can be obtained; it is observed that RR_1 is a function of k, ϕ, n, α, and 'a '. To observe the behavior of $\hat{\theta}_{ST_1}$, we have taken several values of these viz k = 0.1 (0.1)…1.0, ϕ = 0.2 (0.2)…1.6, α = 1%, 5%, 10%, n = 5, 8, 10 and a = ±1, ±2, ±3 ,'a' is the prime important factor and decides about the seriousness of over/under estimation in the real life situation. Similarly, we define the Relative Risk of $\hat{\theta}_{ST_2}$ with respect to $\bar{x}$ under L(Δ) as follows

$$RR_2 = \frac{R_E(\bar{x})}{R(\hat{\theta}_{ST_2})} \tag{4.4}$$

Using (4.2) and (3.4) the expression for RR_2 given in (4.4) can be obtained, it is observed that RR_2 is a function of ϕ, n, α and 'a', it no longer depends on 'k'. To observe the behavior of $\hat{\theta}_{ST_2}$, we have taken several values of these, same as in the case of $\hat{\theta}_{ST_1}$ except the values of 'k'. Some of the graphs of RR_1 and RR_2 for the data considered above are provided in the appendix. However, our conclusions based on all the graphs are given in the next section.

5. Conclusions

As mentioned earlier $\hat{\theta}_{ST_1}$ depends on 'k' also we have taken, k = 0.2 (0.2)… 0.8 and it is observed that $\hat{\theta}_{ST_1}$ performs better than the conventional estimator for almost the whole range of k. The performance is best at k = 0.2, n = 8, for a = -1, however as 'k' increases to k = 0.4, there is a sudden change and the performance improves at a = 1 (positive) and the same trend remains for a = 2 and 3 but the range of Ø changes. It may be stated that for smaller weights a negative value of 'a' is suggested however for higher weights positive value particularly a = 3 should be used. We have taken α = 5% and α = 10% also, it is observed that the $\hat{\theta}_{ST_1}$ still performs better for these values of α^s, but the magnitude of relative risk is maximum at α = 1% out of the three values of α, so α = 1% is the recommended level of significance. As regards the choice of degree of asymmetry 'a' no fixed pattern is observed for various values of 'k' i.e. for some values of 'k', positive 'a' and for some values of 'k' (particularly lower), negative values of 'a' are recommended (say a = -1 for k = 0.2). Looking at the different values of 'a' for different choice of 'k' it seems more logical to remove the arbitrariness in the choice of 'k'. $\hat{\theta}_{ST_2}$ removes this arbitrariness and our conclusions for $\hat{\theta}_{ST_2}$ are as follows:

There will be too many tables for varying 'k',' Ø', 'α', and 'a' all the tables are not presented here.

- For small n = 5 and for different levels of significance considered here $\hat{\theta}_{ST_2}$ performs better than the usual estimator in the whole range of Ø. However, its performance is best for a = ±3, (still better for a = 3) and α = 1%. Hence it is recommended to use the proposed estimator for the positive values of 'a' and small values of ‘n’. Similar results hold for n = 8 and 10 however the magnitude of RR is maximum for n = 8.

- For α = 5% and for n = 5, 8, 10 and for 0.2 ≤ Ø ≤ 1.6, the magnitude of relative risk is still higher, i.e. usual estimate has more risk under L(Δ) compared to $\hat{\theta}_{ST_1}$. Again, $\hat{\theta}_{ST_2}$ performs better for positive values of “a”, as the magnitude of relative risk values is higher it implies better risk control in this situation.

- For α = 10%, rest of the findings are same, i.e., values of n considered here, range of Ø (0.2 ≤ Ø ≤ 1.6) and a = ±1, ±2, ±3. But comparing the values of relative risks for varying α ^{s} (the level of significance) ; It is observed that the magnitude of these values is maximum for α = 1% and a = 1 for all the values as “n” considered here and for 0.2 ≤ Ø ≤ 1.6 as evident from the following table.

So, it is recommended to use $\hat{\theta}_{ST_1}$ for (n=8):

α = 1%	a = 3,	0.2 ≤ Ø ≤ 1.6
α = 5%	a = 3,	0.2 ≤ Ø ≤ 1.6
α = 10%	a = 3,	0.2 ≤ Ø ≤ 1.6

However, it performs well for other values of ‘n’ and ‘a’ also, considered here, but for the above values its performance is at its best.

Finally, use $\hat{\theta}_{ST_2}$ for n = 8, α = 1%, a = +3 and 0.2 ≤ Ø ≤ 1.6. Use $\hat{\theta}_{ST_1}$ for different values of ‘k’ and different values of ‘a’ at α = 1%.
A shrinkage testimator, with shrinkage factor dependent of test statistic has been proposed, above numerical values of relative risks demonstrate its superiority over the usual estimator.

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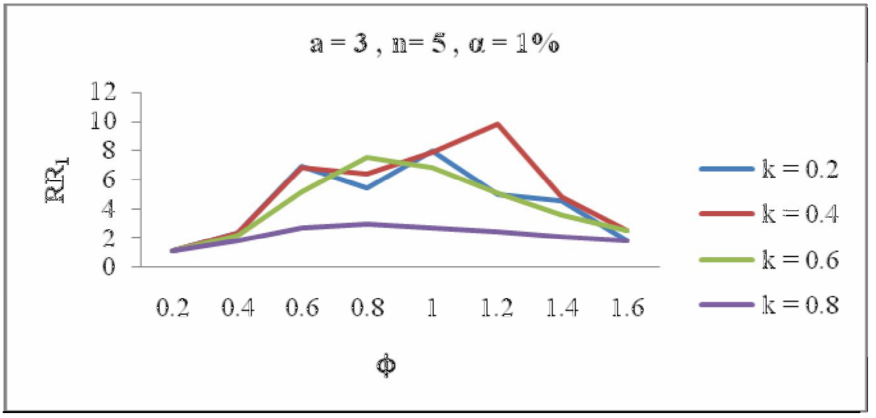
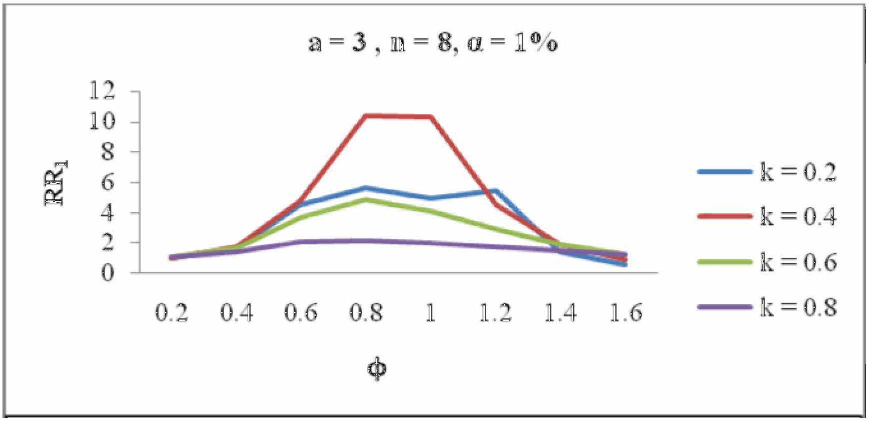
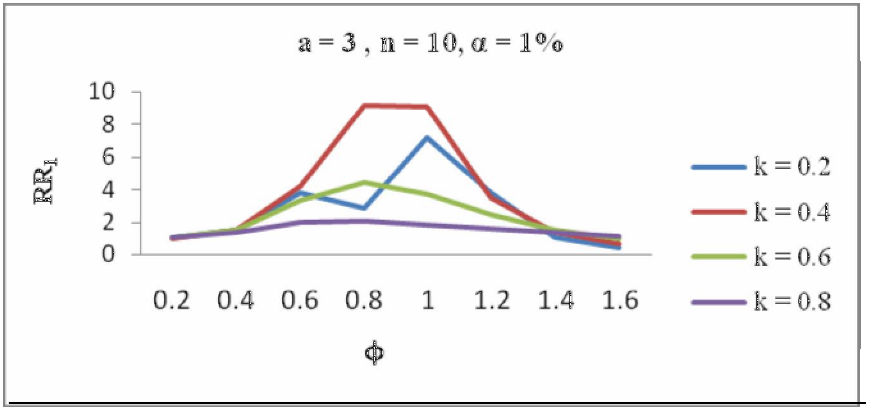
Appendix

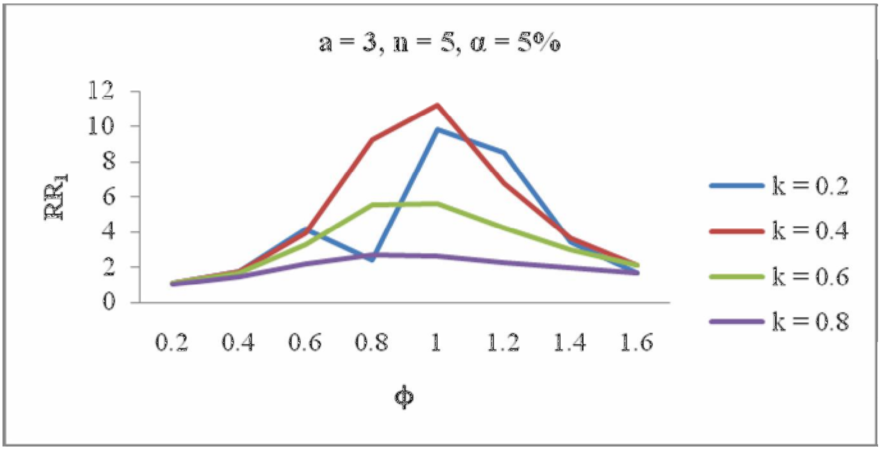
Graphs of Relative Risk for $\hat{\theta}_{ST_1}$ and $\hat{\theta}_{ST_2}$ with respect to conventional estimator.

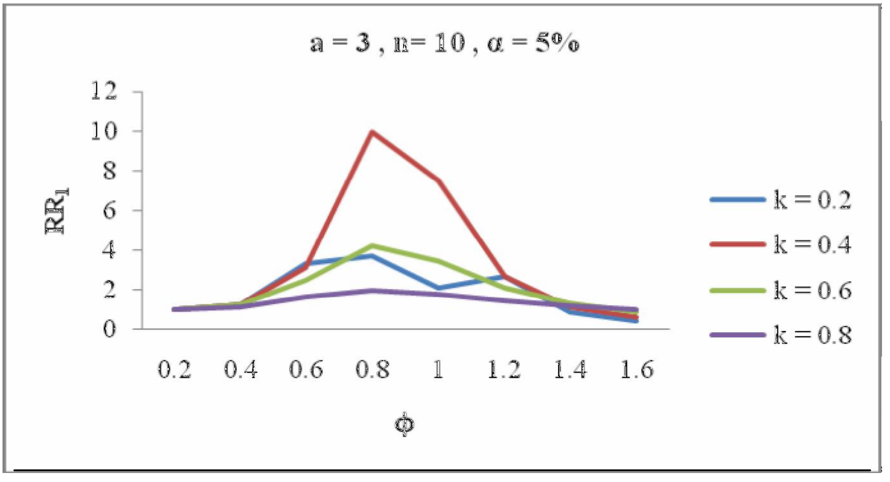
Graphs of Relative Risk for $\hat{\theta}_{ST_1}$

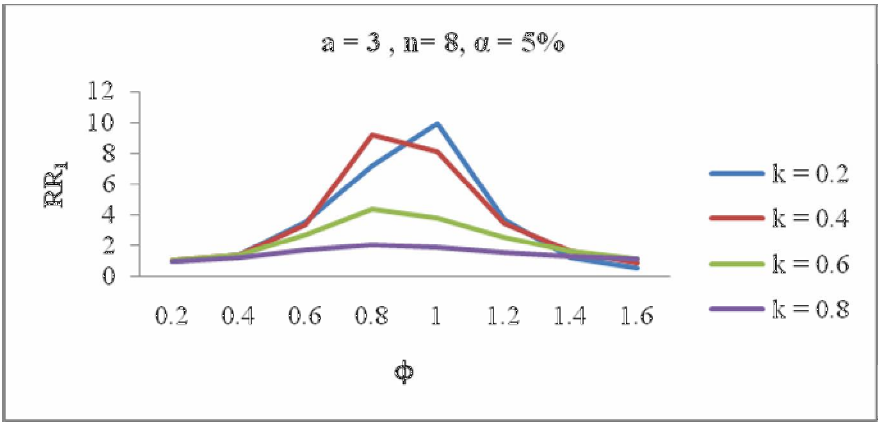


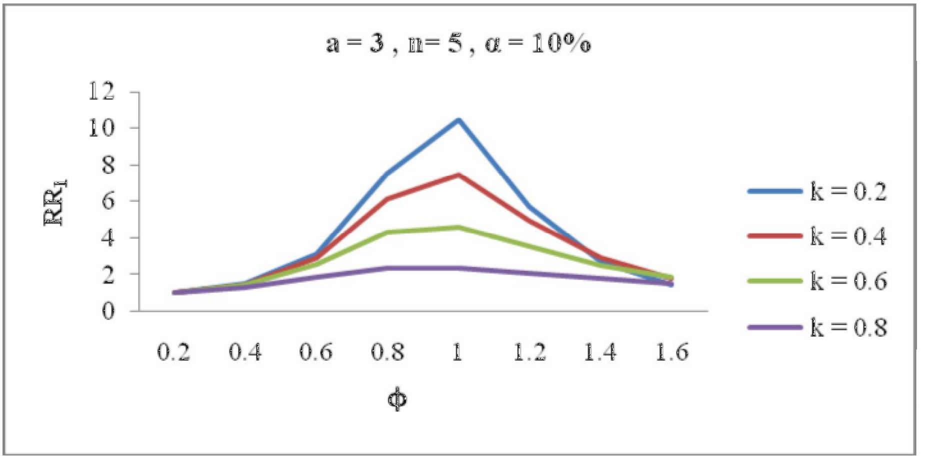


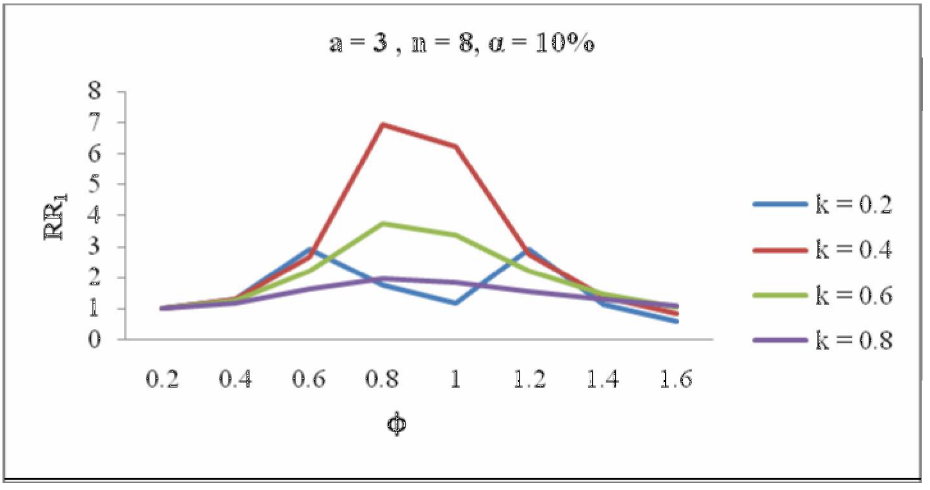


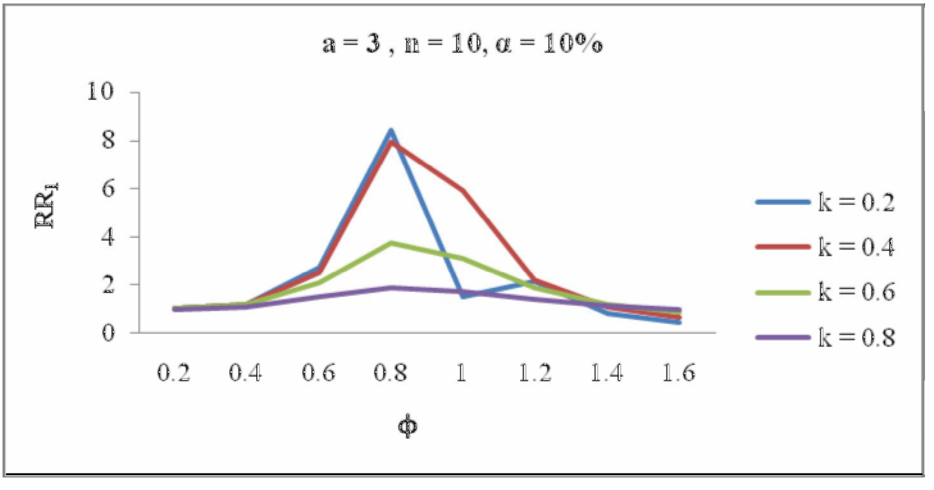

a = 3, n = 5, α = 5%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Φ
k = 0.2
k = 0.4
k = 0.6
k = 0.8

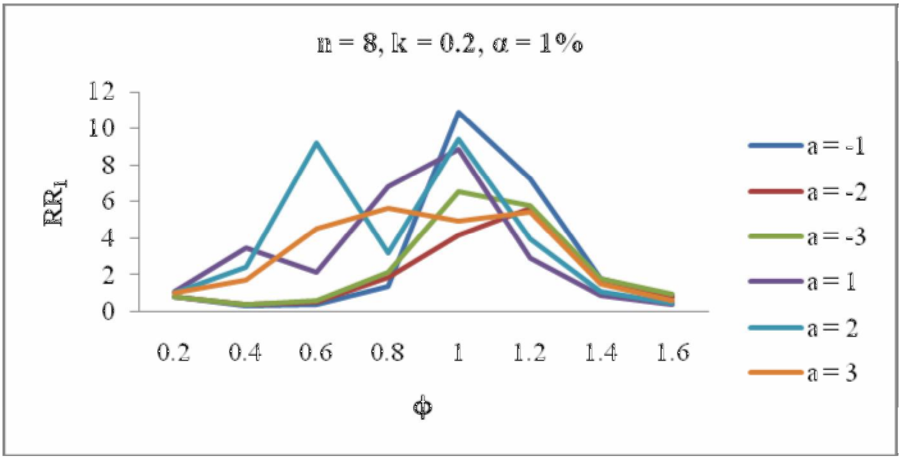

a = 3 , n= 10 , α = 5%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Φ
k = 0.2
k = 0.4
k = 0.6
k = 0.8

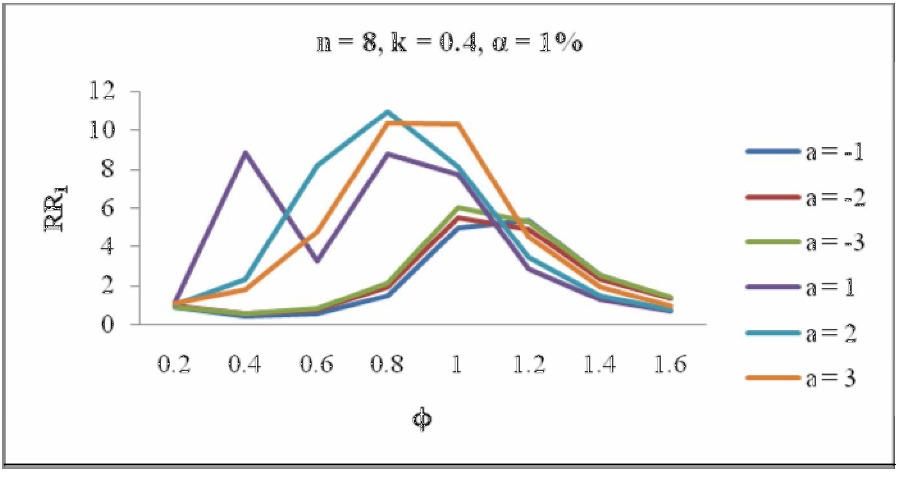

a = 3 , n= 8, α = 5%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Φ
k = 0.2
k = 0.4
k = 0.6
k = 0.8

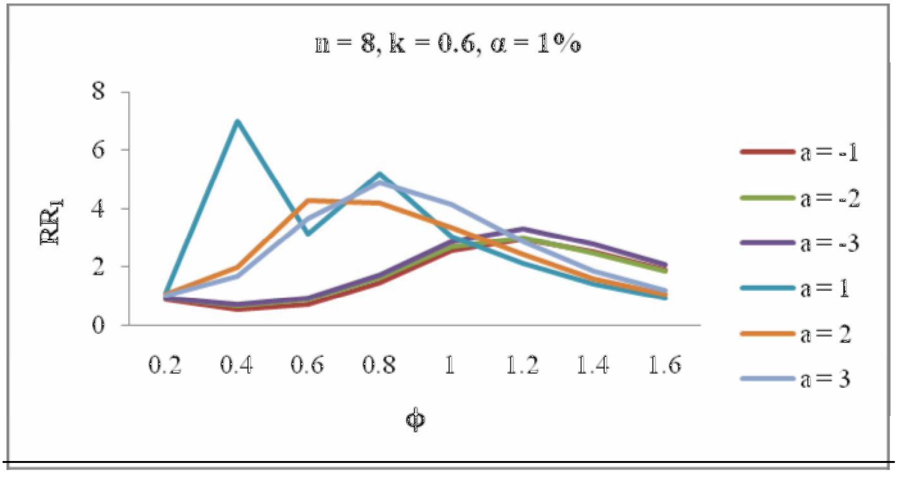

a = 3 , n= 5 , α = 10%
12
10
8
6
4
2
0
RR1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8

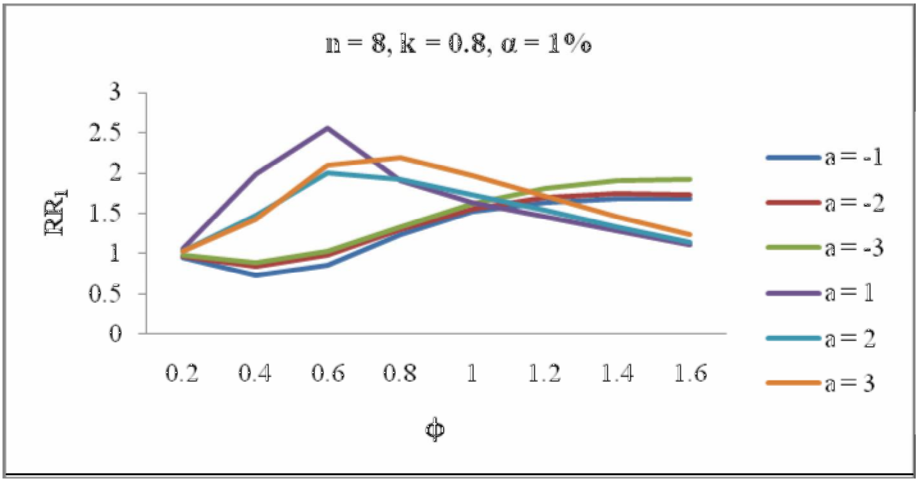

a = 3 , n = 8, α = 10%
8
7
6
5
4
3
2
1
0
RR1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8


a = 3 , n = 10, α = 10%
10
8
6
4
2
0
RR1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8

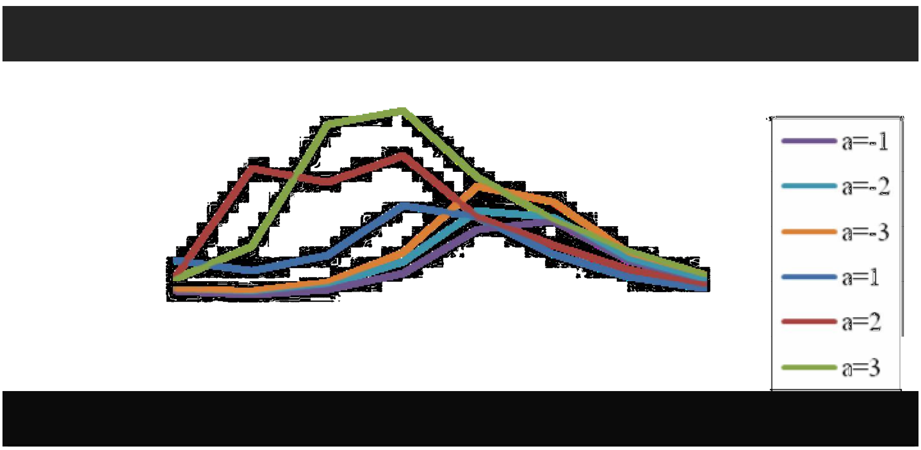

n = 8, k = 0.2, α = 1%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

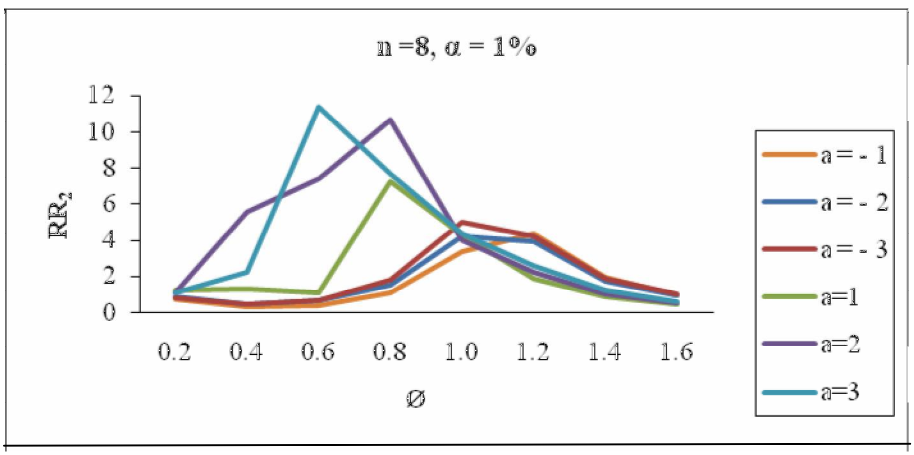

n = 8, k = 0.4, α = 1%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

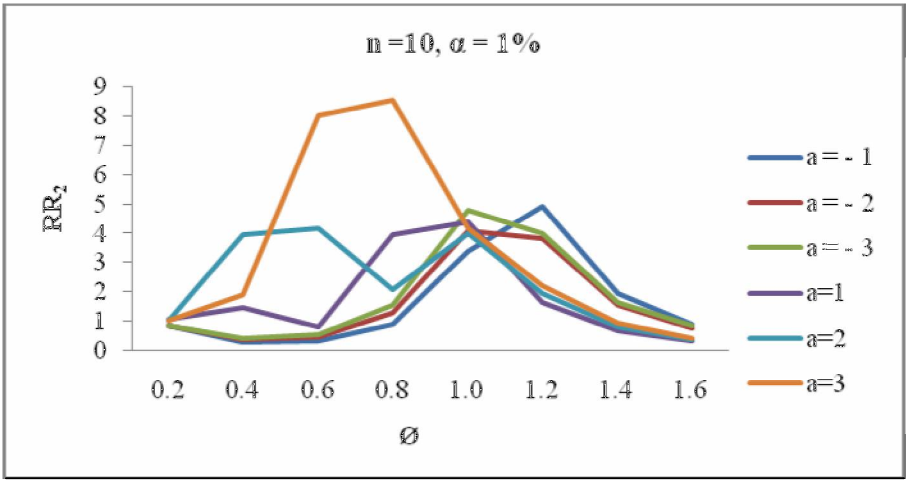

n = 8, k = 0.6, α = 1%
RR1
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

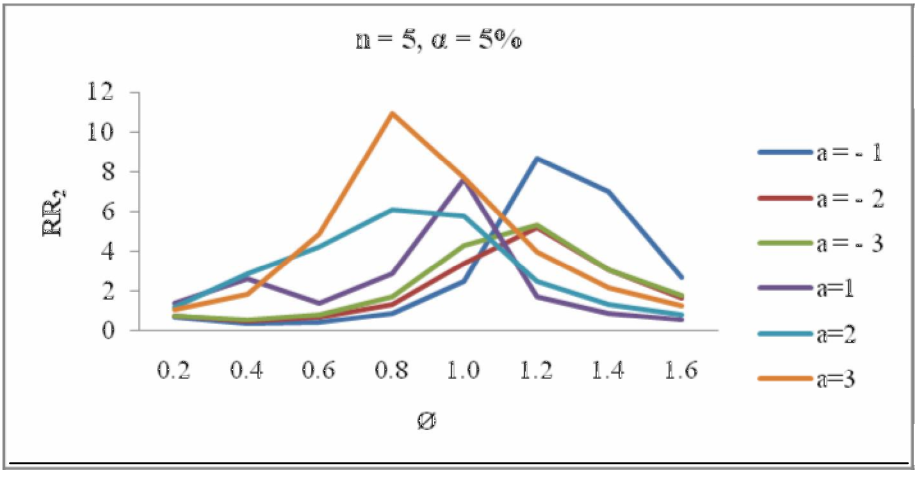


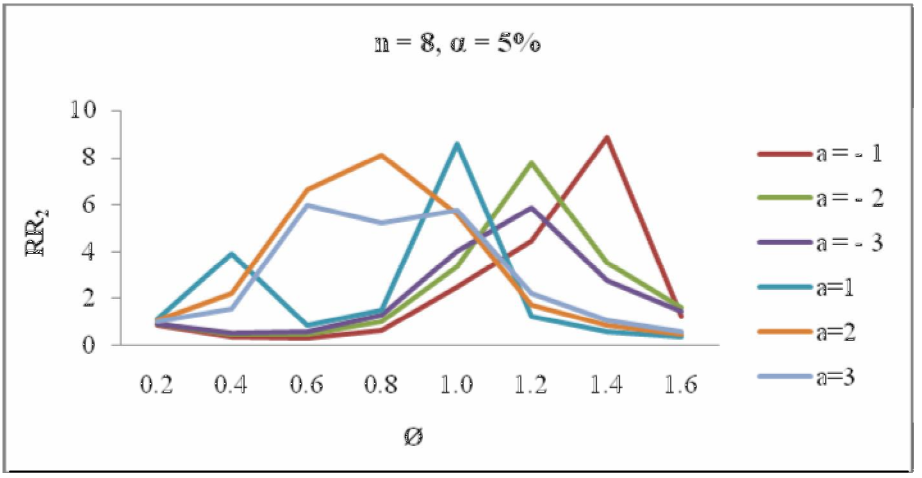
Graphs of Relative Risk for $\hat{\theta}_{ST_2}$

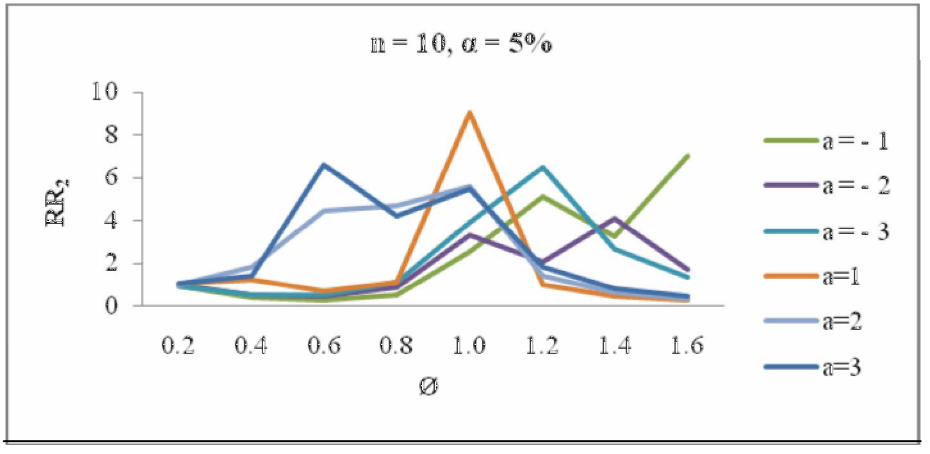


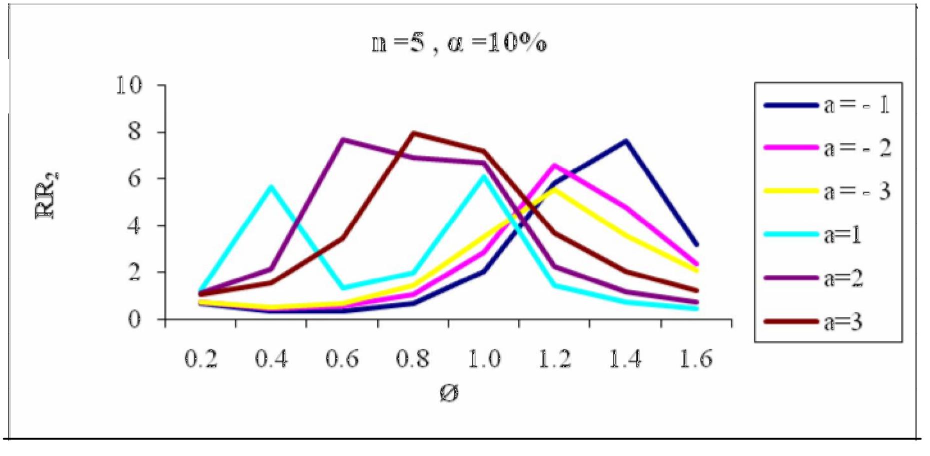


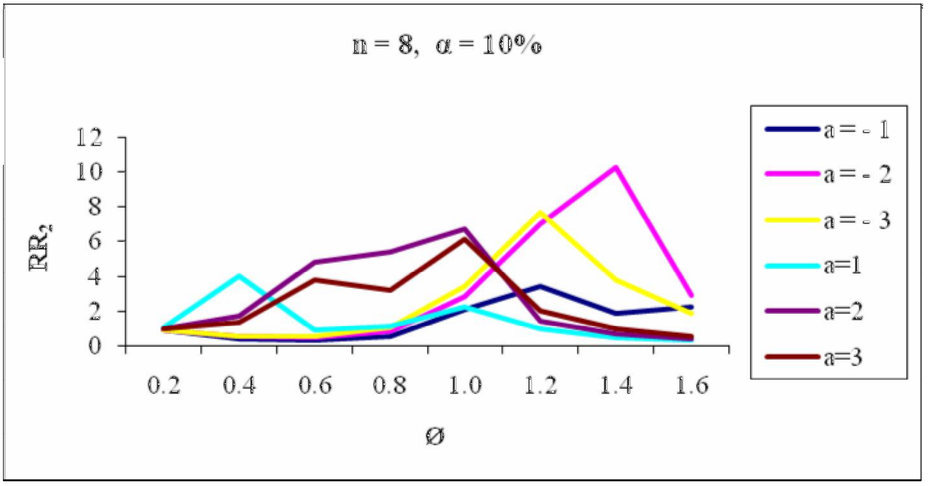

n =10, α = 1%
RR2
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3

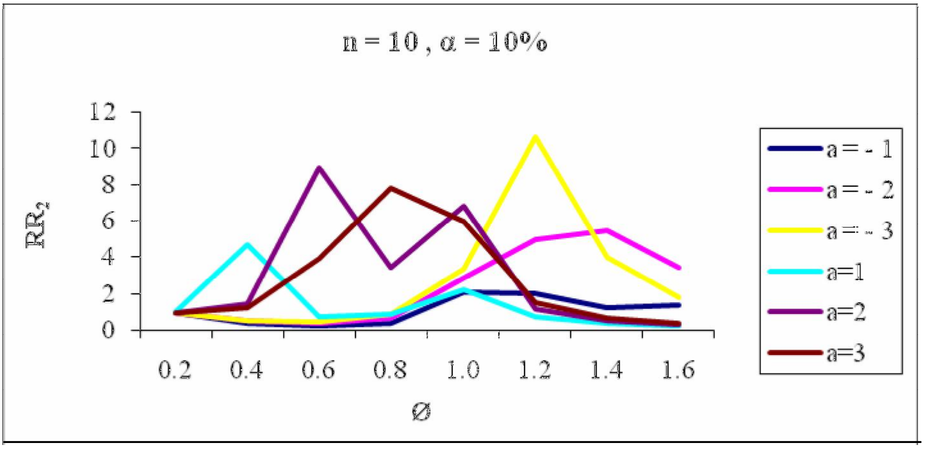

n = 5, α = 5%
RR2
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 8, α = 5%
RR2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 10, α = 5%
RR2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n =5 , α =10%
RR2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 8, α = 10%
RR2
Ø
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0
2
4
6
8
10
12
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 10 , α = 10%
RR2
Ø
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0
2
4
6
8
10
12
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3